Nuclear electric dipole moment of three-body system

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Abstract

Nuclear electric dipole moments of ^{3}He and ^{3}H are calculated using Time Reversal Invariance Violating (TRIV) potentials based on the meson exchange theory, as well as the ones derived by using pionless and pionful effective field theories, with nuclear wave functions obtained by solving Faddeev equations in configuration space for the complete Hamiltonians comprising both TRIV and realistic strong interactions. The obtained results are compared with the previous calculations of ^{3}He EDM and with time reversal invariance violating effects in neutron-deuteron scattering.

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I. INTRODUCTION

The electric dipole moment (EDM) of particles is a very important parameter in searching for Time Reversal Invariance Violation (TRIV) and for the possible manifestation of new physics. The discovery of non-zero value of the EDM would be a clear evidence of TRIV [1], therefore, it has been a subject for intense experimental and theoretical investigations for more than 50 years. The search for TRIV also has fundamental importance for the explanation of the baryon asymmetry [2] of the Universe which requires a source of CP-violation [3] beyond that entering the Cabibbo-Kobayashi- Maskawa matrix of the Standard Model. Any observation of EDM in the near future will be a direct indication of new physics beyond the Standard Model. However, theoretical estimates for the values of particle EDMs are extremely small which results in many difficulties in experimental search for neutron and electron EDMs. Therefore, it is desirable to consider more complex systems, where EDMs or another TRIV parameters could be enhanced. This also would provide assurance that there would be enough observations to avoid a possible "accidental" cancelation of T-violating effects due to unknown structural factors related to strong interactions. The study of the EDM is particularly important for the simplest few-nucleon systems, the result of which may lead to a better understanding of TRIV effects in heavier nuclei. Moreover, few-nucleon systems meet requirements for a number of proposals to measure EDMs of light nuclei in storage rings [4–7].

In this paper, we calculate nuclear EDMs of ^{3}He and ^{3}H using TRIV potential in meson exchange model, as well as in pionless and pionful EFT. Weak TRIV potentials are used in conjunction with realistic strong interaction Hamiltonians. Several realistic nucleon-nucleon potentials have been tested to represent the strong interaction: the Argonne v18(AV18), the Reid soft core(Reid93), the Nijmegen(NijmII), the INOY, as well as the AV18 in conjunction with the three-nucleon Urbanna IX(UIX) potential. Three-nucleon wave functions have been obtained by solving Faddeev equations in the configuration space for the complete Hamiltonians, comprising both TRIV and strong interactions.

II. TIME REVERSAL VIOLATING POTENTIALS

The most general form of time reversal violating and parity violating part of nucleon-nucleon Hamiltonian in the first order of relative nucleon momentum can be written as the sum of momentum independent and momentum dependent parts, $H^{\not{T}\not{P}} = H^{\not{T}\not{P}}_{stat} + H^{\not{T}\not{P}}_{non-static}$ [8],

$$\begin{aligned} H^{\not{T}\not{P}}_{stat} &= g_1(r)\boldsymbol{\sigma}_- \cdot \hat{r} + g_2(r)\tau_1 \cdot \tau_2 \boldsymbol{\sigma}_- \cdot \hat{r} + g_3(r) T^z_{12} \boldsymbol{\sigma}_- \cdot \hat{r} \\ &+ g_4(r)\tau_+ \boldsymbol{\sigma}_- \cdot \hat{r} + g_5(r)\tau_- \boldsymbol{\sigma}_+ \cdot \hat{r} \end{aligned} \tag{1}$$

$$\begin{aligned} H^{\not{T}\not{P}}_{non-static} &= \left(g_6(r) + g_7(r)\tau_1 \cdot \tau_2 + g_8(r) T^z_{12} + g_9(r)\tau_+\right) \boldsymbol{\sigma}_\times \cdot \frac{\bar{\boldsymbol{p}}}{m_N} \\ &+ \left(g_{10}(r) + g_{11}(r)\tau_1 \cdot \tau_2 + g_{12}(r) T^z_{12} + g_{13}(r)\tau_+\right) \\ &\times \left(\hat{r} \cdot \boldsymbol{\sigma}_\times \hat{r} \cdot \frac{\bar{\boldsymbol{p}}}{m_N} - \frac{1}{3}\boldsymbol{\sigma}_\times \cdot \frac{\bar{\boldsymbol{p}}}{m_N}\right) \\ &+ g_{14}(r)\tau_- \left(\hat{r} \cdot \boldsymbol{\sigma}_1 \hat{r} \cdot (\boldsymbol{\sigma}_2 \times \frac{\bar{\boldsymbol{p}}}{m_N}) + \hat{r} \cdot \boldsymbol{\sigma}_2 \hat{r} \cdot (\boldsymbol{\sigma}_1 \times \frac{\bar{\boldsymbol{p}}}{m_N})\right) \\ &+ g_{15}(r)(\tau_1 \times \tau_2)^z \boldsymbol{\sigma}_+ \cdot \frac{\bar{\boldsymbol{p}}}{m_N} \\ &+ g_{16}(r)(\tau_1 \times \tau_2)^z \left(\hat{r} \cdot \boldsymbol{\sigma}_+ \hat{r} \cdot \frac{\bar{\boldsymbol{p}}}{m_N} - \frac{1}{3}\boldsymbol{\sigma}_+ \cdot \frac{\bar{\boldsymbol{p}}}{m_N}\right), \end{aligned} \tag{2}$$

where the exact form of $g_i(r)$ depends on the details of the particular theory. Because of the additional factor, the non-static potential contributions are suppressed by a factor $\frac{\bar{\boldsymbol{p}}}{m_N}$, therefore, we consider here only static TRIV interactions which could be obtained within three different approaches: in a meson exchange model, pionless EFT, and pionful EFT.

The TRIV meson exchange potential in general involves exchanges of pions ($J^P = 0^-$, $m_\pi = 138$ MeV), η-mesons($J^P = 0^-$, $m_\eta = 550$ MeV), and ρ- and ω-mesons ($J^P = 1^-$, $m_{\rho,\omega} = 770, 780$ MeV). To derive this potential, one can use strong $\mathcal{L}^{st}$ and TRIV $\mathcal{L}_{\not{T}\not{P}}$ Lagrangians, which can be written as [9, 10]

$$\begin{aligned} \mathcal{L}^{st} &= g_\pi \bar{N} i\gamma_5 \tau^a \pi^a N + g_\eta \bar{N} i \gamma_5 \eta N \\ &- g_\rho \bar{N}\left(\gamma^\mu - i\frac{\chi_V}{2m_N}\sigma^{\mu\nu} q_\nu\right)\tau^a \rho^a_\mu N \\ &- g_\omega \bar{N}\left(\gamma^\mu - i\frac{\chi_S}{2m_N}\sigma^{\mu\nu} q_\nu\right)\omega_\mu N, \end{aligned} \tag{3}$$

$$
\begin{aligned}
\mathcal{L}_{\not T \not P} &= \bar{N}[\bar{g}^{(0)}_{\pi}\tau^a\pi^a + \bar{g}^{(1)}_{\pi}\pi^0 + \bar{g}^{(2)}_{\pi}(3\tau^z\pi^0 - \tau^a\pi^a)]N \\
&+ \bar{N}[\bar{g}^{(0)}_{\eta}\eta + \bar{g}^{(1)}_{\eta}\tau^z\eta]N \\
&+ \bar{N}\frac{1}{2m_N}[\bar{g}^{(0)}_{\rho}\tau^a\rho^a_{\mu} + \bar{g}^{(1)}_{\rho}\rho^0_{\mu} + \bar{g}^{(2)}(3\tau^z\rho^0_{\mu} - \tau^a\rho^a_{\mu})]\sigma^{\mu\nu}q_{\nu}\gamma_5 N \\
&+ \bar{N}\frac{1}{2m_N}[\bar{g}^{(0)}_{\omega}\omega_{\mu} + \bar{g}^{(1)}_{\omega}\tau^z\omega_{\mu}]\sigma^{\mu\nu}q_{\nu}\gamma_5 N, \qquad (4)
\end{aligned}
$$

where $q_\nu = p_\nu - p'_\nu$, χ_V and χ_S are iso-vector and scalar magnetic moments of a nucleon ($\chi_V = 3.70$ and $\chi_S = -0.12$), and $\bar{g}^{(i)}_{\alpha}$ are TRIV meson-nucleon coupling constants.

Then, a TRIV potential obtained from these Lagrangians can be written as

$$
\begin{aligned}
V_{\not T \not P} &= \left[-\frac{\bar{g}^{(0)}_{\eta}g_\eta}{2m_N}\frac{m^2_\eta}{4\pi}Y_1(x_\eta) + \frac{\bar{g}^{(0)}_{\omega}g_\omega}{2m_N}\frac{m^2_\omega}{4\pi}Y_1(x_\omega)\right]\boldsymbol{\sigma}_-\cdot\hat{r} \\
&+ \left[-\frac{\bar{g}^{(0)}_{\pi}g_\pi}{2m_N}\frac{m^2_\pi}{4\pi}Y_1(x_\pi) + \frac{\bar{g}^{(0)}_{\rho}g_\rho}{2m_N}\frac{m^2_\rho}{4\pi}Y_1(x_\rho)\right]\tau_1\cdot\tau_2\boldsymbol{\sigma}_-\cdot\hat{r} \\
&+ \left[-\frac{\bar{g}^{(2)}_{\pi}g_\pi}{2m_N}\frac{m^2_\pi}{4\pi}Y_1(x_\pi) + \frac{\bar{g}^{(2)}_{\rho}g_\rho}{2m_N}\frac{m^2_\rho}{4\pi}Y_1(x_\rho)\right]T^z_{12}\boldsymbol{\sigma}_-\cdot\hat{r} \\
&+ \left[-\frac{\bar{g}^{(1)}_{\pi}g_\pi}{4m_N}\frac{m^2_\pi}{4\pi}Y_1(x_\pi) + \frac{\bar{g}^{(1)}_{\eta}g_\eta}{4m_N}\frac{m^2_\eta}{4\pi}Y_1(x_\eta) + \frac{\bar{g}^{(1)}_{\rho}g_\rho}{4m_N}\frac{m^2_\rho}{4\pi}Y_1(x_\rho) + \frac{\bar{g}^{(1)}_{\omega}g_\omega}{4m_N}\frac{m^2_\omega}{4\pi}Y_1(x_\omega)\right]\tau_+\boldsymbol{\sigma}_-\cdot\hat{r} \\
&+ \left[-\frac{\bar{g}^{(1)}_{\pi}g_\pi}{4m_N}\frac{m^2_\pi}{4\pi}Y_1(x_\pi) - \frac{\bar{g}^{(1)}_{\eta}g_\eta}{4m_N}\frac{m^2_\eta}{4\pi}Y_1(x_\eta) - \frac{\bar{g}^{(1)}_{\rho}g_\rho}{4m_N}\frac{m^2_\rho}{4\pi}Y_1(x_\rho) + \frac{\bar{g}^{(1)}_{\omega}g_\omega}{4m_N}\frac{m^2_\omega}{4\pi}Y_1(x_\omega)\right]\tau_-\boldsymbol{\sigma}_+\cdot\hat{r}, \qquad (5)
\end{aligned}
$$

where $T^z_{12} = 3\tau^z_1\tau^z_2 - \tau_1\cdot\tau_2$, $Y_1(x) = (1+\frac{1}{x})\frac{e^{-x}}{x} = -\frac{d}{dx}Y_0(x)$, $Y_0(x) = \frac{e^{-x}}{x}$,$x_a = m_a r$.

Comparing eq. (1) with this potential, one can see that $g_i(r)$ functions in a meson exchange model can be identified as

$$
\begin{aligned}
g^{ME}_1(r) &= -\frac{\bar{g}^{(0)}_{\eta}g_\eta}{2m_N}\frac{m^2_\eta}{4\pi}Y_1(x_\eta) + \frac{\bar{g}^{(0)}_{\omega}g_\omega}{2m_N}\frac{m^2_\omega}{4\pi}Y_1(x_\omega) \\
g^{ME}_2(r) &= -\frac{\bar{g}^{(0)}_{\pi}g_\pi}{2m_N}\frac{m^2_\pi}{4\pi}Y_1(x_\pi) + \frac{\bar{g}^{(0)}_{\rho}g_\rho}{2m_N}\frac{m^2_\rho}{4\pi}Y_1(x_\rho) \\
g^{ME}_3(r) &= -\frac{\bar{g}^{(2)}_{\pi}g_\pi}{2m_N}\frac{m^2_\pi}{4\pi}Y_1(x_\pi) + \frac{\bar{g}^{(2)}_{\rho}g_\rho}{2m_N}\frac{m^2_\rho}{4\pi}Y_1(x_\rho) \\
g^{ME}_4(r) &= -\frac{\bar{g}^{(1)}_{\pi}g_\pi}{4m_N}\frac{m^2_\pi}{4\pi}Y_1(x_\pi) + \frac{\bar{g}^{(1)}_{\eta}g_\eta}{4m_N}\frac{m^2_\eta}{4\pi}Y_1(x_\eta) + \frac{\bar{g}^{(1)}_{\rho}g_\rho}{4m_N}\frac{m^2_\rho}{4\pi}Y_1(x_\rho) + \frac{\bar{g}^{(1)}_{\omega}g_\omega}{4m_N}\frac{m^2_\omega}{4\pi}Y_1(x_\omega) \\
g^{ME}_5(r) &= -\frac{\bar{g}^{(1)}_{\pi}g_\pi}{4m_N}\frac{m^2_\pi}{4\pi}Y_1(x_\pi) - \frac{\bar{g}^{(1)}_{\eta}g_\eta}{4m_N}\frac{m^2_\eta}{4\pi}Y_1(x_\eta) - \frac{\bar{g}^{(1)}_{\rho}g_\rho}{4m_N}\frac{m^2_\rho}{4\pi}Y_1(x_\rho) + \frac{\bar{g}^{(1)}_{\omega}g_\omega}{4m_N}\frac{m^2_\omega}{4\pi}Y_1(x_\omega), \qquad (6)
\end{aligned}
$$

The TRIV potentials in pionless EFT (without explicit pion contributions) contain only point-like nucleon-nucleon interactions which are proportional to the local delta functions. Thus, one can write the corresponding $g_i(r)$ functions as

$$g^{\not\pi}_{i=1..5}(r) = \frac{c^{\not\pi}_{i=1..5}}{2m_N}\frac{d}{dr}\delta^{(3)}(r) \to \frac{c^{\not\pi}_{i=1..5}\mu^2}{2m_N}\left(-\frac{\mu^2}{4\pi}Y_1(\mu r)\right), \tag{7}$$

where low energy constants (LECs) $c^{\not\pi}_i$ of pionless EFT have the dimension of $[fm^2]$. Here, for numerical calculations we approximate the singular delta functions by the Yukawa type functions as $\delta^{(3)}(r) \simeq \frac{\mu^3}{4\pi}Y_0(\mu r)$, as it was done in [10], with a natural scale of the parameter $\mu \simeq m_\pi$.

In the pionful EFT, the long range terms of the potential are due to the one pion exchange whereas the short range terms are similar to the ones obtained within the pionless EFT. Then, by ignoring the contribution of the two pion exchange at the middle range scale, as well as other higher order corrections, one can write $g_i(r)$ functions for the pionful EFT as

$$\begin{aligned}
g^\pi_1(r) &= -\frac{c^\pi_1\mu^2}{2m_N}\frac{\mu^2}{4\pi}Y_1(\mu r) \\
g^\pi_2(r) &= -\frac{c^\pi_2\mu^2}{2m_N}\frac{\mu^2}{4\pi}Y_1(\mu r) - \frac{\bar{g}^{(0)}_\pi g_\pi}{2m_N}\frac{m_\pi^2}{4\pi}Y_1(x_\pi) \\
g^\pi_3(r) &= -\frac{c^\pi_3\mu^2}{2m_N}\frac{\mu^2}{4\pi}Y_1(\mu r) - \frac{\bar{g}^{(2)}_\pi g_\pi}{2m_N}\frac{m_\pi^2}{4\pi}Y_1(x_\pi) \\
g^\pi_4(r) &= -\frac{c^\pi_4\mu^2}{2m_N}\frac{\mu^2}{4\pi}Y_1(\mu r) - \frac{\bar{g}^{(1)}_\pi g_\pi}{4m_N}\frac{m_\pi^2}{4\pi}Y_1(x_\pi) \\
g^\pi_5(r) &= -\frac{c^\pi_5\mu^2}{2m_N}\frac{\mu^2}{4\pi}Y_1(\mu r) - \frac{\bar{g}^{(1)}_\pi g_\pi}{4m_N}\frac{m_\pi^2}{4\pi}Y_1(x_\pi).
\end{aligned} \tag{8}$$

For this potential, the cutoff scale μ is larger than pion mass, because pion is an explicit degree of freedom of the theory.[1] The following identities might be useful in order to compare this potential with the one used in reference [11]:

$$\frac{g_\pi}{2m_N} \leftrightarrow \frac{g_A}{F_\pi} \text{ in [11]}, \quad \bar{g}^{(0,1)}_\pi \leftrightarrow \left(-\frac{\bar{g}_{0,1}}{F_\pi}\right) \text{ in [11]}, \quad \frac{c^\pi_{1,2}}{2m_N} \leftrightarrow \frac{1}{2}\bar{C}_{1,2} \text{ in [11]}. \tag{9}$$

[1] These expressions do not include the cutoff for the pion exchange terms. The introduction of this cutoff with the corresponding Fourier transformation will modify the Yukawa functions. This will modify the contributions from the short distance terms, as well as the form of the scalar functions in contact terms. However, these corrections are of a higher order. It should be mentioned that the values of the LECs and their scaling behavior, as a function of a cutoff parameter $c^\pi_i(\mu)$, differ from the corresponding behavior of $c^{\not\pi}_i(\mu)$ terms obtained in pionless EFT.

However, the parameters $c^{\pi}_{3,4,5}$ and $\bar{g}^{(2)}_{\pi}$ were not included at the leading order potential in [11] because they were considered as higher order terms with additional assumptions related to the source of TRIV interactions. In this paper, we calculate contributions from all the operators without making any assumption about the possible value of the coefficients for each operator.

It is important that all these three potentials which come from different approaches have exactly the same operator structure. Thus, the only difference between them is related to the difference in corresponding scalar functions which, in turn, differ only by the values of the characteristic masses: m_π, m_η, m_ρ, and m_ω. Therefore, to unify the notation, it is convenient to define the new constants C^a_n (having dimension of $[fm]$) together with the scalar function $f^a_n(r) = \frac{\mu^2}{4\pi} Y_1(\mu r)$ (having dimension of $[fm^{-2}]$) as

$$g_n(r) \equiv \sum_a C^a_n f^a_n(r), \tag{10}$$

where the expressions of C^a_n and $f^a_n(r)$ are provided in eqs. (6), (7), and (8).

Since the non-static TRIV potential terms, with $g_{n>5}$, do not appear either in a meson exchange model or in the lowest order EFTs, they can be considered as a higher order correction to the lowest order EFT or be related to heavy meson or multi-meson contributions in the meson exchange model.

III. ^{3}He AND ^{3}H EDMS

The value of nuclear EDM is defined as

$$d = \langle JJ|\hat{D}|JJ\rangle = \sqrt{\frac{J}{(2J+1)(J+1)}} \langle J||\hat{D}||J\rangle, \tag{11}$$

where $|JJ\rangle$ is a nuclear wave function with a total spin and its projection equal to J. The EDM operator $\hat{D}$ contains direct contributions from the intrinsic nucleon EDMs (current operators)

$$\hat{D}^{nucleon}_{\not{T}\not{P}} = \sum_i \frac{1}{2}[(d_p + d_n) + (d_p - d_n)\tau^z_i]\boldsymbol{\sigma}_i \tag{12}$$

and contributions from the nuclear EDM polarization operator

$$\hat{D}^{pol}_{PT} = \sum_i Q_i \boldsymbol{r}_i, \tag{13}$$

which describe polarization of the nuclei due to TRIV potentials. Here, d_n and d_p are neutron and proton EDMs, and Q_i and r_i are charge and position of i-th nucleon. In this work, we do not consider possible TRIV meson exchange current contributions. Therefore, the value of ^{3}He (or ^{3}H) EDM can be expressed as

$$d = \frac{1}{\sqrt{6}} \left[\langle\Psi||\hat{D}^{nucleon}_{\not{T}\not{P}}||\Psi\rangle + \langle\Psi_{\not{T}\not{P}}||\hat{D}^{pol}_{TP}||\Psi\rangle + \langle\Psi||\hat{D}^{pol}_{TP}||\Psi_{\not{T}\not{P}}\rangle \right], \tag{14}$$

where $|\Psi\rangle$ and $|\Psi_{\not{T}\not{P}}\rangle$ represent the time reversal invariant and TRIV parts of the nuclear wave function.

First, we will analyze the contribution of the intrinsic nucleon EDM momenta to the nuclear EDM which are defined by the action of operator defined in eq.(12) on the nuclear wave function. For the deuteron (the two-body system), the contributions of the intrinsic neutron and proton EDMs simply add $d_d^{nucleon} = d_p + d_n$, i.e. this value does not depend on the nuclear wave function and, consequently, on the choice of the particular strong interaction model. The situation is different in the three-body system, ^{3}H or ^{3}He nuclei, where the EDM contributions from intrinsic nucleon EDMs become wave function dependent. One can see that nucleonic contributions to the nuclear EDMs are in rather good agreement for all strong potentials with local interactions: the AV18, the Reid93 and the Nijm II. Nevertheless, these values differ for the models that includ non-locality or for the ones where a three-nucleon force is added (see Table I where EDM calculations from the references [11, 12] are also listed). Softer two-nucleon interaction models[2] have a tendency to provide the nuclear EDM values closer to the ones of the unpaired nucleon (i.e. neutronic EDM for ^{3}He case, and protonic EDM for ^{3}H). The addition of the three-nucleon force provides an effect similar to the one of hardening the interaction. This effect is clearly related to the strength of the tensor force, which permits one to reduce the pairing of the nucleons in the three-nucleon system. Let us mention that our results for this single-nucleonic operator are in excellent agreement with those from references [11, 12].

[2] The INOY interaction model has the strongest non-locality and is the softest one from the interactions considered in the table I, followed by the EFT and the CD-BONN potentials, respectively.

TABLE I: The nucleon electric dipole moment contributions to nuclear EDMs calculated for different strong interaction potentials.

model	^{3}He	^{3}H
AV18	$-0.0468d_p + 0.877d_n$	$0.877d_p - 0.0480d_n$
Reid93	$-0.0465d_p + 0.878d_n$	$0.879d_p - 0.0475d_n$
NijmII	$-0.0458d_p + 0.880d_n$	$0.880d_p - 0.0468d_n$
AV18UIX	$-0.0542d_p + 0.868d_n$	$0.868d_p - 0.0552d_n$
INOY	$-0.0229d_p + 0.927d_n$	$0.928d_p - 0.0236d_n$
CD-BONN[11, 12]	$-0.0370d_p + 0.897d_n$	-
AV18[11, 12]	$-0.0470d_p + 0.877d_n$	-
EFT NN[11, 12]	$-0.0310d_p + 0.905d_n$	-
EFT NN+NNN[11, 12]	$-0.0350d_p + 0.901d_n$	-

In order to calculate polarization contributions to the nuclear EDM, we solve Faddeev equations in a configuration space [13] by including TRIV potentials. We consider neutrons and protons as isospin-degenerate states of the same particle nucleon whose mass is fixed to $\hbar^2/m = 41.471$ MeV·fm. By using the isospin formalism, the three Faddeev equations become formally identical, which for pairwise interactions reads

$$(E - H_0 - V_{ij})\,\psi_k = V_{ij}(\psi_i + \psi_j), \tag{15}$$

where (ijk) are particle indexes, H_0 is the kinetic energy operator, V_{ij} is a two body force between particles i, and j, and $\psi_k = \psi_{ij,k}$ is the so-called Faddeev component. In the last equation, the potential formally contains both a strong interaction (TRI conserving) part (V_{ij}^{TC}) and a TRIV (parity violating) part ($V^{\not{T}\not{P}}$), i.e.: $V_{ij} = V_{ij}^{TC} + V_{ij}^{\not{T}\not{P}}$. Due to the presence of TRIV potential, the system's wave function does not have a definite parity and contains both positive and negative parity components. As a consequence, the Faddeev components of the total wave function can be split into the sum of positive- and negative-parity parts:

$$\psi_k = \psi_k^+ + \psi_k^-. \tag{16}$$

Three-nucleon bound state wave function has a strongly predominant positive-parity component. The TRIV interaction is weak ($V_{ij}^{\not{T}\not{P}} << V_{ij}^{TC}$). Then, by neglecting second-order

terms in TRIV potential, one obtains a system of two differential equations:

$$\left(E - H_0 - V_{ij}^{TC}\right)\psi_k^+ \; = \; V_{ij}^{TC}(\psi_i^+ + \psi_j^+), \tag{17}$$

$$\left(E - H_0 - V_{ij}^{TC}\right)\psi_k^- \; = \; V_{ij}^{TC}(\psi_i^- + \psi_j^-) + V_{ij}^{\not T \not P}(\psi_i^+ + \psi_j^+ + \psi_k^+) \tag{18}$$

One can see that the first equation (17) defines only the positive-parity part of the wave function. This equation contains only a strong nuclear potential and corresponds to the standard three-nucleon problem: a bound state of helium or triton. The solution of the second differential equation (18), which contains an inhomogeneous term $V_{ij}^{\not T \not P}(\psi_i^+ + \psi_j^+ + \psi_k^+)$, gives us the negative-parity components of the wave functions.

To solve these equations numerically, we use our standard procedure described in detail in [14]. Using a set of Jacobi coordinates, defined by $\boldsymbol{x}_k = (\boldsymbol{r}_j - \boldsymbol{r}_i)$ and $\boldsymbol{y}_k = \frac{2}{\sqrt{3}}(\boldsymbol{r}_k - \frac{\boldsymbol{r}_i + \boldsymbol{r}_j}{2})$, we expand each Faddeev component of the wave function in bipolar harmonic basis:

$$\psi_k^{\pm} = \sum_{\alpha} \frac{F_\alpha^{\pm}(x_k, y_k)}{x_k y_k} \left| \left(l_x \left(s_i s_j\right)_{s_x}\right)_{j_x} \left(l_y s_k\right)_{j_y} \right\rangle_{JM} \otimes \left| (t_i t_j)_{t_x} t_k \right\rangle_{TT_z}, \tag{19}$$

where index α represents all allowed combinations of the quantum numbers presented in the brackets, l_x and l_y are the partial angular momenta associated with respective Jacobi coordinates, and s_i and t_i are spins and isospins of the individual particles. Functions $F_\alpha(x_k, y_k)$ are called partial Faddeev amplitudes. In the expansion (19), we consider both possible total isospin channels $T = 1/2$ and $T = 3/2$, regardless of the fact that positive-parity components ψ_k^+ have predominant contribution of $T = 1/2$ state.

Equations (17) and (18) must be supplemented the appropriate boundary conditions for Faddeev partial amplitudes $F_\alpha^{\pm}$: partial Faddeev amplitudes are regular at the origin

$$F_\alpha^{\pm}(0, y_k) = F_\alpha^{\pm}(x_k, 0) = 0, \tag{20}$$

and the system's wave function vanishes exponentially as either x_k or y_k becomes large. This condition is imposed by setting Faddeev amplitudes to vanish at the borders (x_{max}, y_{max}) of a chosen grid, i.e.:

$$F_\alpha^{\pm}(x_k, y_{max}) = 0, \quad F_\alpha^{\pm}(x_{max}, y_k) = 0. \tag{21}$$

This formalism can be easily generalized to accommodate three-nucleon forces, as is described in paper [15].

In Table II, we summarize the calculation of the matrix elements $\frac{2}{\sqrt{6}}\langle\Psi||\hat{D}_{TP}^{pol}||\Psi_{\not T \not P}\rangle$; values for each TRIV operator from Eq.(5) obtained for a different choice of the strong interaction

are tabulated. In this table, operators of TRIV potential (5) are calculated when combined with a unified scalar function

$$- \frac{1}{2m_N}\frac{\Lambda^2}{4\pi}Y_1(\Lambda r) \tag{22}$$

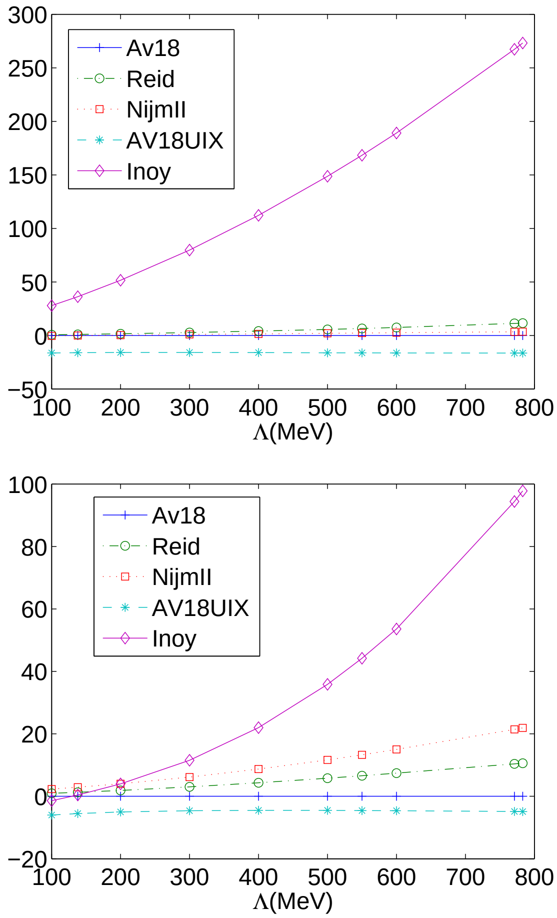
and calculated when taking various values of the parameter Λ, which was chosen to coincide with the masses of π, η, ρ and ω mesons. Therefore, this table can be used to analyze the TRIV potentials in the meson exchange model, whereas once multiplied by an additional Λ^2 cutoff factor, it also can be used for the TRIV potentials in pionless EFT or pionful EFT.

TABLE II: Contribution of the different TRIV operators (Eq.(5) to the expectation value of $\frac{2}{\sqrt{6}}\langle\Psi||\hat{D}^{pol}_{TP}||\Psi_{\not{T}\not{P}}\rangle$. Calculations has been performed for several different strong potentials and for ^{3}He (^{3}He) nucleus; values are given in 10^{-3} e-fm units.

operator	Λ	AV18	Reid93	NijmII	AV18UIX	INOY
1	m_π	−5.32(5.28)	−5.37(5.33)	−5.31(5.28)	−4.46(4.42)	−7.24(7.23)
	m_η	−0.571(0.572)	−0.608(0.609)	−0.584(0.585)	−0.478(0.477)	−1.53(1.54)
	m_ρ	−0.233(0.234)	−0.26(0.261)	−0.241(0.242)	−0.195(0.195)	−0.857(0.862)
	m_ω	−0.223(0.224)	−0.249(0.25)	−0.231(0.232)	−0.187(0.186)	−0.833(0.838)
2	m_π	5.9(−5.89)	6.08(−6.07)	6.12(−6.11)	5.5(−5.48)	10.3(−10.2)
	m_η	0.673(−0.681)	0.803(−0.81)	0.771(−0.777)	0.629(−0.635)	2.72(−2.73)
	m_ρ	0.292(−0.296)	0.387(−0.391)	0.351(−0.354)	0.27(−0.273)	1.6(−1.6)
	m_ω	0.281(−0.284)	0.374(−0.378)	0.337(−0.341)	0.259(−0.262)	1.56(−1.56)
3	m_π	6.76(−7.02)	6.78(−7.01)	6.76(−6.98)	6.66(−6.89)	7.46(−7.72)
	m_η	0.775(−0.814)	0.773(−0.804)	0.762(−0.794)	0.784(−0.819)	1.25(−1.31)
	m_ρ	0.304(−0.32)	0.3(−0.312)	0.295(−0.307)	0.308(−0.322)	0.645(−0.674)
	m_ω	0.29(−0.305)	0.285(−0.297)	0.281(−0.293)	0.294(−0.307)	0.625(−0.653)
4	m_π	2.17(2.42)	2.2(2.41)	2.25(2.46)	2.81(3.03)	2.27(2.48)
	m_η	0.286(0.319)	0.291(0.317)	0.296(0.322)	0.372(0.403)	0.397(0.436)
	m_ρ	0.112(0.125)	0.114(0.125)	0.116(0.127)	0.146(0.159)	0.202(0.223)
	m_ω	0.107(0.12)	0.109(0.119)	0.111(0.121)	0.139(0.152)	0.196(0.216)
5	m_π	19.4(19.6)	19.6(19.8)	20(20.2)	18.3(18.5)	19.5(19.6)
	m_η	2.43(2.47)	2.59(2.63)	2.75(2.8)	2.32(2.35)	3.5(3.56)
	m_ρ	0.985(1.01)	1.09(1.11)	1.2(1.22)	0.937(0.953)	1.92(1.95)
	m_ω	0.942(0.961)	1.04(1.06)	1.15(1.17)	0.896(0.911)	1.86(1.9)

Similar to the intrinsic nucleon EDM contribution, presented in table I, dynamical nuclear EDMs are also rather insensitive to the choice of the local strong interaction potential. The addition of the three-nucleon force affects the results by 10-20%, whereas the presence of the strong non-locality in two-nucleon interaction (as in the INOY model) has a very large impact. This fact is also confirmed by the cutoff dependence behavior of the relative deviations of the $\frac{2}{\sqrt{6}}\langle\Psi||\hat{D}^{pol}_{TP}||\Psi_{\not{T}\not{P}}\rangle$ matrix element for operators 1 and 5 calculated for

FIG. 1: The relative deviations of the $d^{pol}_{^3\mathrm{He}}$ value from the one obtained for AV18 potential $\frac{d^{pol}-d^{pol}(AV18)}{d^{pol}(AV18)} \times 100$. Results are presented for the operators 1(upper) and 5(lower) and as a function of the cutoff parameter.



different strong interaction potentials in relation to the matrix element calculated with AV18 potential (see Fig. (1)), where the first plot corresponds to the operator 1 and second one to the operator 5.

Using meson exchange TRIV potential from Eq.(5), we can present the results of the calculations of nuclear EDMs as the sum of contributions of the different TRIV potential terms to the $\frac{2}{\sqrt{6}}\langle\Psi||\hat{D}^{pol}_{TP}||\Psi_{\not{T}\not{P}}\rangle$ matrix element, as is shown in Table III.

TABLE III: Contributions to $\frac{2}{\sqrt{6}}\langle\Psi||\hat{D}^{pol}_{TP}||\Psi_{\not{T}\not{P}}\rangle$ for ^{3}He(^{3}H) EDMs from different terms of meson exchange TRIV potential in 10^{-3} e-fm units. We use the following values for strong couplings constants: $g_\pi = 13.07, \quad g_\eta = 2.24, \quad g_\rho = 2.75, \quad g_\omega = 8.25$. A similar table can be inferred for the case of pionless and pionful EFT from Table II.

Couplings	AV18	Reid93	NijmII	AV18UIX	INOY	AV18[12]
$\bar{g}^0_\pi$	77.2(-76.9)	79.5(-79.3)	80.0(-79.8)	71.9(-71.6)	134(-134)	157
$\bar{g}^1_\pi$	141(144)	143(145)	145(148)	138(141)	142(145)	288
$\bar{g}^2_\pi$	88.3(-91.8)	88.7(-91.6)	88.3(-91.3)	87.1(-90.1)	98.5(-102)	444
$\bar{g}^0_\rho$	-0.803(0.814)	-1.06(1.08)	-0.964(0.974)	-0.742(0.751)	-4.40(4.41)	-1.65
$\bar{g}^1_\rho$	1.20(1.21)	1.34(1.35)	1.49(1.50)	1.09(1.09)	2.36(2.37)	2.48
$\bar{g}^2_\rho$	-0.836(0.879)	-0.824(0.858)	-0.811(0.845)	-0.846(0.885)	-1.77(1.85)	4.13
$\bar{g}^0_\omega$	1.84(-1.85)	2.05(-2.06)	1.91(-1.91)	1.54(-1.54)	6.88(-6.91)	4.13
$\bar{g}^1_\omega$	-4.33(-4.46)	-4.74(-4.86)	-5.19(-5.32)	-4.27(-4.38)	-8.49(-8.71)	-9.08
$\bar{g}^0_\eta$	-1.28(1.28)	-1.36(1.36)	-1.31(1.31)	-1.07(1.07)	-3.43(3.45)	-
$\bar{g}^1_\eta$	2.40(2.41)	2.57(2.59)	2.75(2.77)	2.18(2.18)	3.48(3.50)	-

The last column in Table III shows the results for ^{3}He EDM obtained in reference [12]. Comparing results of [12] with our calculations for AV18 potential, one can see that there is a systematic discrepancy for all the values of the matrix elements. For these calculations, we have used the same strong and TRIV potential as in [12]. It, therefore, points at the possible systematic error in one of the calculations. We use solutions of the Faddeev equations, while the calculations of the wave functions in [12] have been done in a no-core shell model framework using perturbative expansion for the negative parity states. Based on the presented results, it is impossible to figure out if the discrepancy is the result of a numerical error in one of the algorithms or if there is an intrinsic limitation of the perturbative expansion used in reference [12] with no-core shell model approach[3].

On the other hand, our calculations for the deuteron (two-body system) using the same

[3] Curiously enough, our results differ from the ones of reference [12] roughly by the factor 2 for all the isospin rank-0 and rank-1 TRIV potential terms, whereas isospin rank-2 operator results differ by factor 5.

formalism and employing AV18 strong interaction gives

$$d_d^{(pol)} = 18.95 \times 10^{-2} \bar{g}_\pi^1 + 3.52 \times 10^{-3} \bar{g}_\eta^1 + 17.13 \times 10^{-4} \bar{g}_\rho^1 - 49.09 \times 10^{-4} \bar{g}_\omega^1, \tag{23}$$

which is in excellent agreement with the result of reference [10]

$$d_d^{(pol)} = 18.69 \times 10^{-2} \bar{g}_\pi^1 + 3.56 \times 10^{-3} \bar{g}_\eta^1 + 17.19 \times 10^{-4} \bar{g}_\rho^1 - 49.17 \times 10^{-4} \bar{g}_\omega^1. \tag{24}$$

IV. CONCLUSIONS

Using the data from Table III, one can obtain the polarization parts of ^{3}He and ^{3}H EDMs for the choice of AV18UIX strong potential. Then, the expressions for ^{3}He and ^{3}H EDMs can be written as

$$\begin{aligned} d_{^3\mathrm{He}} \; = \; & (-0.0542 d_p + 0.868 d_n) + 0.072[\bar{g}_\pi^{(0)} + 1.92\bar{g}_\pi^{(1)} + 1.21\bar{g}_\pi^{(2)} - 0.015\bar{g}_\eta^{(0)} + 0.03\bar{g}_\eta^{(1)} \\ & -0.010\bar{g}_\rho^{(0)} + 0.015\bar{g}_\rho^{(1)} - 0.012\bar{g}_\rho^{(2)} + 0.021\bar{g}_\omega^{(0)} - 0.06\bar{g}_\omega^{(1)}] e \cdot fm \end{aligned} \tag{25}$$

and

$$\begin{aligned} d_{^3\mathrm{H}} \; = \; & (0.868 d_p - 0.0552 d_n) - 0.072[\bar{g}_\pi^{(0)} - 1.97\bar{g}_\pi^{(1)} + 1.26\bar{g}_\pi^{(2)} - 0.015\bar{g}_\eta^{(0)} - 0.030\bar{g}_\eta^{(1)} \\ & -0.010\bar{g}_\rho^{(0)} - 0.015\bar{g}_\rho^{(1)} - 0.012\bar{g}_\rho^{(2)} + 0.022\bar{g}_\omega^{(0)} + 0.061\bar{g}_\omega^{(1)}] e \cdot fm. \end{aligned} \tag{26}$$

It should be noted that in general neutron and proton EDMs can not be related to the meson-nucleon TRIV constants from TRIV potential. However, it is convenient to present expressions for these EDMs, obtained in the chiral limit [16] with the assumption that nucleon EDM are resulted from TRIV potential

$$d_n = -d_p = \frac{e}{m_N} \frac{g_\pi(\bar{g}_\pi^{(0)} - \bar{g}_\pi^{(2)})}{4\pi^2} \ln \frac{m_N}{m_\pi} \simeq 0.14(\bar{g}_\pi^{(0)} - \bar{g}_\pi^{(2)}), \tag{27}$$

which could be used for some models of CP-violation.

Finally, one can compare the obtained expressions for nuclear EDMs with TRIV effects in neutron deuteron elastic scattering related to the $\boldsymbol{\sigma}_n \cdot (\boldsymbol{p} \times \boldsymbol{I})$ correlation, where $\boldsymbol{\sigma}_n$ is the neutron spin, $\boldsymbol{I}$ is the target spin, and $\boldsymbol{p}$ is the neutron momentum, which can be observed in the transmission of polarized neutrons through a target with polarized nuclei. This correlation leads to the difference [17] between the total neutron cross sections for $\boldsymbol{\sigma}_n$ parallel and anti-parallel to $\boldsymbol{p} \times \boldsymbol{I}$

$$\Delta\sigma_{\not{T}\not{P}} = \frac{4\pi}{p} \mathrm{Im}(f_+ - f_-), \tag{28}$$

and to neutron spin rotation angle [18] ϕ around the axis $\boldsymbol{p}\times\boldsymbol{I}$

$$\frac{d\phi_{\not T \not P}}{dz} = -\frac{2\pi N}{p}\mathrm{Re}(f_+ - f_-). \qquad (29)$$

Here, $f_{+,-}$ are the zero-angle scattering amplitudes for neutrons polarized parallel and anti-parallel to the $\boldsymbol{p}\times\boldsymbol{I}$ axis, respectively, z is the target length, and N is the number of target nuclei per unit volume. Using results of [19], one can write

$$\begin{aligned}\frac{1}{N}\frac{d\phi^{\not T \not P}}{dz} &= (-65\ \mathrm{rad}\cdot\ \mathrm{fm}^2)[\bar{g}_\pi^{(0)} + 0.12\bar{g}_\pi^{(1)} + 0.0072\bar{g}_\eta^{(0)} + 0.0042\bar{g}_\eta^{(1)} \\ &\quad -0.0084\bar{g}_\rho^{(0)} + 0.0044\bar{g}_\rho^{(1)} - 0.0099\bar{g}_\omega^{(0)} + 0.00064\bar{g}_\omega^{(1)}]\end{aligned} \qquad (30)$$

and

$$\begin{aligned}P^{\not T \not P} = \frac{\Delta\sigma^{\not T \not P}}{2\sigma_{tot}} &= \frac{(-0.185\ \mathrm{b})}{2\sigma_{tot}}[\bar{g}_\pi^{(0)} + 0.26\bar{g}_\pi^{(1)} - 0.0012\bar{g}_\eta^{(0)} + 0.0034\bar{g}_\eta^{(1)} \\ &\quad -0.0071\bar{g}_\rho^{(0)} + 0.0035\bar{g}_\rho^{(1)} + 0.0019\bar{g}_\omega^{(0)} - 0.00063\bar{g}_\omega^{(1)}].\end{aligned} \qquad (31)$$

One can see that both nuclear EDMs and elastic scattering TRIV effects are mostly sensitive to TRIV pion coupling constants. However, while the EDM values are equally sensitive to all isospin parts of the pion coupling constant, the elastic scattering effects are mainly defined by the isoscalar interactions. This fact clearly demonstrates the complementarity of different TRIV effects in three-nucleon systems. Thus, the relative values of these TRIV parameters may vary for differen models of CP-violation and, therefore, the measurement of a number of TRIV observables can help to avoid a possible accidental cancelation of TRIV .

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